Author(s) : K.S. Kölbig
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Submitter :
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Zero of a Function of One Real Variable

Function subprograms RZEROX and DZEROX compute, to an attempted specified accuracy, a zero x_0 of a real-valued function $f(x)$ lying in a given interval $[a, b]$, where $f(a) * f(b) \leq 0$.

On computers other than CDC or Cray, only the double precision version DZEROX is available. On CDC and Cray computers, only the single-precision version RZEROX is available.

Structure:

FUNCTION subprograms
User Entry Names: RZEROX, DZEROX
Obsolete User Entry Names: ZEROX $\equiv$ RZEROX
Files Referenced: Unit 6
External References: User-supplied FUNCTION subprogram

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

```
tZEROX(A,B,EPS,MAXF,F,MODE)
```

has, in any arithmetic expression, the value x_0.

A,B — (type according to t) On entry, A and B must specify the end points of the search interval. Unchanged on exit.

EPS — (type according to t) On entry, EPS must be equal to the accuracy parameter (see **Accuracy**). Unchanged on exit.

MAXF — (INTEGER) On entry, MAXF must be equal to the maximum permitted number of references to the function F within the iteration loop. Unchanged on exit.

F — (type according to t) Name of a user-supplied FUNCTION subprogram, declared EXTERNAL in the calling program. This subprogram must set F(X) $= f($X$)$.

MODE — (INTEGER) On entry, MODE = 1 or MODE = 2 defines the algorithm for finding x_0 (see **Method** and **Notes**).

Method:

Two algorithims are incorporated in this subprogram. These are described in Ref. 1 as “Algorithm M” (MODE = 1) and “Algorithm R” (MODE = 2). Both are mixtures of linear interpolation, rational interpolation and bisection.

Accuracy:

These subprograms try to compute two numbers x_0 and x_1 lying in the interval $[a, b]$ such that

1. $f(x_0)f(x_1) \leq 0$
2. $|f(x_0)| \leq |f(x_1)|$
3. $|x_0 - x_1| \leq 2 * \mathtt{EPS} * (1 + |x_0|)$

If successful, the value of x_0 is assigned to the function name.

Notes:

1. $\texttt{MODE} = \texttt{1}$ should be used for fairly simple functions whose evaluation is cheap in comparison with the calculations performed in one iteration step of `RZEROX` or `DZEROX`.
2. $\texttt{MODE} = \texttt{2}$ should be used for more expensive functions. Convergence should be faster than for $\texttt{MODE} = \texttt{1}$, but the evaluation steps are more expensive.
3. For functions which have a pole near the exact zero, $\texttt{MODE} = \texttt{1}$ is recommended because of the special character of the interpolation formula which is used.

Error handling:

1. $\texttt{F(A)} * \texttt{F(B)} > 0$. The function value is set equal to zero.
2. `MODE` has a value other than `1` or `2`. The function value is set equal to zero.
3. The number of references to `F` exceeds `MAXF`. The function value is set equal to the last computed value of x_0 (see **Accuracy**)

For each error a message is printed.

Source:

The subprogram is based on Algol programs described in Ref. 1.

References:

1. J.C.P. Bus and T.J. Dekker, Two efficient algorithms with garanteed convergence for finding a zero of a function, ACM Trans. Math. Software **1** (1975) 330–345.

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