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Submitter :
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Library: MATHLIB
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Riemann Zeta Function

Function subprograms `RRIZET` and `DRIZET` calculate the Riemann zeta function

$$\zeta(x) \;=\; \sum_{k=1}^{\infty} k^{-x} \;=\; \frac{1}{\Gamma(x)} \int_0^{\infty} \frac{t^{x-1}}{e^t - 1} dt \qquad (x > 1)$$

for real arguments $x \neq 1$, where $\zeta(x)$ is defined by analytic continuation for $x < 1$. For $x = 1$, $\zeta(x)$ has a pole of order one.

On CDC and Cray computers, the double-precision version `DRIZET` is not available.

Structure:

`FUNCTION` subprograms
User Entry Names: `RRIZET`, `DRIZET`
Files Referenced: `Unit 6`
External References: `GAMMA` (C302), `DGAMMA` (C302), `MTLMTR` (N002), `ABEND` (Z035)

Usage:

In any arithmetic expression,

`RRIZET(X)` or `DRIZET(X)`

has the value $\zeta($`X`$)$ if `X` < 1, and $\zeta($`X`$) - 1$ if `X` > 1, where `RRIZET` is of type `REAL`, `DRIZET` is of type `DOUBLE PRECISION`, and where `X` has the same type as the function name.

Method:

Rational Chebyshev approximation. For $x < \frac{1}{2}$ the reflection formula for $\zeta(x)$ is used.

Accuracy:

`RRIZET` (except on CDC and Cray computers) has full single-precision accuracy. For most values of the argument `X`, `DRIZET` (and `RRIZET` on CDC and Cray computers) has an accuracy of approximately one significant digit less than the machine precision.

Error handling:

Error `C315.1: X = 1`. The function value is set to zero, and a message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.

References:

1. W.J. Cody, K.E. Hillstrom, and H.C. Thather, Jr., Chebyshev approximations for the Riemann zeta function, Math. Comp. **25** (1971) 537–547.

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