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Whittaker Function M of Complex Argument and Complex Indices

Function subprograms CWHITM and WWHITM compute the Whittacker function

$$M_{\kappa,\mu}(z) \;=\; e^{-\frac{1}{2}z} z^{\frac{1}{2}+\mu} M(\tfrac{1}{2}+\mu-\kappa, 1+2\mu, z)$$

for complex arguments z and complex indices κ, μ, where $M(a,b,z)$ is Kummer's function (See Ref. 1). The z-plane is cut along the negative real axis.

The double-precision version WWHITM is available only on computers which support a COMPLEX*16 Fortran data type.

Structure:

FUNCTION subprograms
User Entry Names: CWHITM, WWHITM
Files Referenced: Unit 6
External References: CLGAMA (C306), WLGAMA (C306), CCLBES (C309), WCLBES (C309), MTLMTR (N002), ABEND (Z035)

Usage:

In any arithmetic expression,

CWHITM(Z,KA,MU) or WWHITM(Z,KA,MU) has the value $M_{\mathtt{KA},\mathtt{MU}}(\mathtt{Z})$,

where KA $= \kappa$ and MU $= \mu$. CWHITM is of type COMPLEX, WWHITM is of type COMPLEX*16, and Z, KA and MU have the same type as the function name.

Method:

For $\mu-\kappa+\frac{1}{2}$ or $\mu+\kappa+\frac{1}{2}$ equal to a negative integer, $M_{\kappa,\mu}(z)$ reduces to a polynomial in z. For other values, a regular Coulomb wave function $F_0(\nu,\rho)$ is computed by using subprogram CCLBES (C309) in conjunction with functional relations.

Restrictions:

$\mu \neq -\frac{1}{2}, -\frac{3}{2}, \ldots$; Re $z \geq 0$ if Im $z = 0$.

Accuracy:

CWHITM (except on CDC and Cray computers) has full single-precision accuracy. For most values of the arguments, WWHITM (and CWHITM on CDC and Cray computers) has an accuracy of approximately two to three decimal digits less than the machine precision.

Error handling:

Error C328.1: $\mathtt{Z} = \mathtt{X} + \mathtt{iY}$ with $\mathtt{X} < 0$ and $\mathtt{Y} = 0$.
Error C328.2: $2 * \mathtt{MU} = -\,n,\ (n = 1, 2, \ldots)$.
In both cases, the function value is set equal to zero, and a message is written on Unit 6, unless subroutine MTLSET (N002) has been called. An error message is also written on Unit 6 if the internal call to CCLBES or WCLBES returns JFAIL $\neq 0$ (see Short write-up for CCLBES (C309)).

References:

1. M. Abramowitz and I.A. Stegun (Eds.), Handbook of Mathematical Functions, Chapter 13, Confluent Hypergeometric Functions, 9th printing with corrections, (Dover, New York 1972).

2. L.J. Slater, Confluent hypergeometric functions, (University Press, Cambridge 1960)

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