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Incomplete Gamma Functions

Function subprograms RGAPNC, DGAPNC and RGAGNC, DGAGNC calculate the incomplete gamma function

$$P(a,x) = \begin{cases} \dfrac{1}{\Gamma(a)} \displaystyle\int_0^x e^{-t}\, t^{a-1}\, dt & (a > 0) \\ e^{-x}\, x^a\, \dfrac{M(1,a+1,x)}{\Gamma(a+1)} & (a \le 0), \end{cases}$$

and the complementary incomplete gamma function

$$G(a,x) = \begin{cases} \dfrac{1}{\Gamma(a)} \displaystyle\int_x^\infty e^{-t}\, t^{a-1}\, dt & (a > 0) \\ e^x\, x^{-a} \displaystyle\int_x^\infty e^{-t}\, t^{a-1}\, dt & (a \le 0), \end{cases}$$

respectively, for real arguments $x \ge 0$ and a. $M(a,b,x)$ is Kummer's function (see Ref. 3).
On CDC and Cray computers, the double-precision versions DGAPNC and DGAGNC are not available.

Structure:

FUNCTION subprograms
Uses Entry Names: RGAPNC, RGAGNC, DGAPNC, DGAGNC
Obsolete User Entry Names: GAPNC ≡ RGAPNC, GAGNC ≡ RGAGNC
Files Referenced: Unit 6
External References: ALGAMA (C304), DLGAMA (C304), MTLMTR (N002), ABEND (Z035)

Usage:

In any arithmetic expression,

RGAPNC(A,X)	or	DGAPNC(A,X)	has the value	$P(\mathtt{A},\mathtt{X})$,
RGAGNC(A,X)	or	DGAGNC(A,X)	has the value	$G(\mathtt{A},\mathtt{X})$,

where RGAPNC and RGAGNC are of type REAL, DGAPNC and DGAGNC are of type DOUBLE PRECISION, A and X have the same type as the function name.

Method:

The method is described in Ref. 1.

Accuracy:

RGAPNC and RGAGNC (except on CDC and Cray computers) have full single-precision accuracy. For most values of the arguments, DGAPNC, DGAGNC (and RGAPNC, RGAGNC on CDC and Cray computers) have an accuracy of approximately two significant digits less than the machine precision.

Restrictions:

For $P(a,x)$: Either (i) $\mathtt{X} > 0$, or (ii) $\mathtt{X} = 0$ and $\mathtt{A} \ge 0$.
For $G(a,x)$: Either (i) $\mathtt{X} > 0$, or (ii) $\mathtt{X} = 0$ and $\mathtt{A} \ne 0$.

Error handling:

Error `C334.1`: `X` $<$ `0`.
Error `C334.2`: For `RGAPNC` and `DGAPNC`: `A` $<$ `0` and `X` $=$ `0`; for `RGAGNC` and `DGAGNC`: `A` $=$ `X` $=$ `0`.
Error `C334.3`: Problems with convergence (unlikely).
In all cases, the function value is set equal to zero, and a message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.

Notes:

When speed is more important than accuracy, e.g. for applications in statistics, use `GAMDIS` (G106) for computing $P(a,x)$. Note, however, that in this case the arguments `A` and `X` must be interchanged.

Source:

The subprograms are based on a Fortran program for the incomplete gamma functions published in Ref. 2.

References:

1. W. Gautschi, A computational procedure for incomplete gamma functions, ACM Trans. Math. Software **5** (1979) 466–481.
2. W. Gautschi, Algorithm 542, Incomplete gamma functions, Collected Algorithms from CACM (1979).
3. M. Abramowitz and I.A. Stegun (Eds.), Handbook of Mathematical Functions, Chapter 13, Confluent Hypergeometric Functions, 9th printing with corrections, (Dover, New York 1972).

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