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Submitter :
Language : Fortran

Library: MATHLIB
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Exponential Integral for Complex Argument

Function subprograms `CEXPIN` and `WEXPIN` calculate the the exponential integral

$$E(z) \;=\; \int_0^z t^{-1}\,(1-e^{-t})\,dt$$

for complex arguments z.
The double-precision version `WEXPIN` is available only on computers which support a `COMPLEX*16` Fortran data type.

Structure:

`FUNCTION` subprograms
Use Entry Names : `CEXPIN`, `WEXPIN`
Files referenced : `Unit 6`
External References: `MTLMTR` (N002), `ABEND` (Z035)

Usage:

In any arithmetic expression,

`CEXPIN(Z)` or `WEXPIN(Z)` has the value $E(\mathtt{Z})$,

where `CEXPIN` is of type `COMPLEX`, `WEXPIN` is of type `COMPLEX*16`, and `Z` has the same type as the function name.

Method:

Padé approximants are used to compute $E(z) = E(x+iy)$ in the following (partly overlapping) regions of the z-plane:

$$\begin{aligned}
&\text{(i)} && (\tfrac{1}{7}(x-1))^2 + (\tfrac{1}{5}y)^2 && \le 1 && (x \ge -5),\\
&\text{(ii)} && (\tfrac{1}{15}(x+12))^2 + (\tfrac{1}{12}y)^2 && \ge 1 && (x \ge -12),\\
&\text{(iii)} && (\tfrac{1}{12}y)^2 && \ge 1 && (x < -12).
\end{aligned}$$

In the remaining region, consisting mainly of a strip along the negative real axis, $E(z)$ is computed by numerical integration (which is very much slower than the evaluation of the Padé approximations).

Accuracy:

`CEXPIN` (except on CDC and Cray computers) has full single-precision accuracy. For most values of the argument `Z`, `WEXPIN` (and `CEXPIN` on CDC and Cray computers) has an accuracy of approximately two significant digits less than the machine precision.

Error handling:

Error `C338.1`: Numerical integration not successful (unlikely). The function value is set equal to zero, and a message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.

References:

1. Y.L. Luke, the special functions and their approximations, v. II, (Academic Press, New York 1969) 198–199, 402–416.

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