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Submitter :
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Modified Bessel Functions I and K of Order 1/3 and 2/3

Function subprograms BSIR3, BSKR3 and DBSIR3, DBSKR3 calculate the modified Bessel functions

$$I_{\nu/3}(x) \quad \text{and} \quad K_{\nu/3}(x)$$

for real arguments $x > 0$ and $\nu = -2, -1, 1, 2$. The value $x = 0$ is permitted for the functions I if $\nu > 0$. Note that the functions K are even with respect to ν.
On CDC and Cray computers, the double-precision versions DBSIR3 etc. are not available.

Structure:

FUNCTION subprograms
User Entry Names: BSIR3, BSKR3, EBSIR3, EBSKR3, DBSIR3, DBSKR3, DEBIR3, DEBKR3
Files Referenced: Unit 6
External References: MTLMTR (N002), ABEND (Z035)

Usage:

In any arithmetic expression,

BSIR3(X,NU)	or	DBSIR3(X,NU)	has the value	$I_{\mathtt{NU}/3}(\mathtt{X})$,
BSKR3(X,NU)	or	DBSKR3(X,NU)	has the value	$K_{\mathtt{NU}/3}(\mathtt{X})$,
EBSIR3(X,NU)	or	DEBIR3(X,NU)	has the value	$\exp(-\mathtt{X}) * I_{\mathtt{NU}/3}(\mathtt{X})$,
EBSKR3(X,NU)	or	DEBKR3(X,NU)	has the value	$\exp(\mathtt{X}) * K_{\mathtt{NU}/3}(\mathtt{X})$,

where BSIR3 etc. are of the type REAL, DBSIR3 etc. are of the type DOUBLE PRECISION, and X has the same type as the function name. NU (INTEGER) has one of the values -2,-1,1,2.

Method:

Approximation by rational functions (I for $|x| < 8$, K for $1 \le x \le 5$), by an algorithm based on power series (K for $0 < x < 1$), or else by truncated Chebyshev series.

Accuracy:

BSIR3 etc. (except on CDC and Cray computers) has full single-precision accuracy. For most values of the argument X, DBSIR3 etc. (and BSIR3 etc. on CDC and Cray computers) has an accuracy of approximately one significant digit less than the machine precision.

Error handling:

Error C340.1: $\mathtt{X} \le 0$ or $\mathtt{X} < 0$, repectively, or $\mathtt{NU} \ne -2, -1, 1, 2$.
The function value is set equal to zero, and a message is written on Unit 6, unless subroutine MTLSET (N002) has been called.

References:

1. Y.L. Luke, Mathematical functions and their approximations (Academic Press, New York 1975) 352, 355, 363, 366.
2. N.M. Temme, On the numerical evaluation of the modified Bessel function of the third kind, J. Comp. Phys. **19** (1975) 324–337.

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