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Submitter :
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Integration over a Triangle

Function subprograms `RTRINT` and `DTRINT` compute an approximate value of the integral

$$I = \int\int f(x,y)\,dxdy,$$

evaluated over the interior of an arbitrary triangle Δ in the xy-plane. An attempted accuracy may, optionally, be specified.

On computers other than CDC or Cray, only the double-precision version `DTRINT` is available. On CDC and Cray computers, only the single-precision version `RTRINT` is available.

Structure:

`FUNCTION` subprograms
User Entry Names: `RTRINT`, `DTRINT`
Obsolete User Entry Names: `TRIINT` $\equiv$ `RTRINT`
Files Referenced: `Unit 6`
External References: `MTLMTR` (N002), `ABEND` (Z035)

Usage:

For `t` = `R` (type `REAL`), `t` = `D` (type `DOUBLE PRECISION`),

```
tTRINT(F,NSD,NPT,EPS,X1,Y1,X2,Y2,X3,Y3)
```

has, in any arithmetic expression, the approximate value of the integral I.

`F` (type according to `t`) Name of a user-supplied `FUNCTION` subprogram, declared `EXTERNAL` in the calling program. This subprogram must set `F(X,Y)` = f`(X,Y)`.

`NSD` (`INTEGER`)
= `0` : No subdivision of the given triangle.
= `1` : Subdivision of the given triangle (see Method).

`NPT` (`INTEGER`)
= `7` : A 7-point integration formula is used.
= `25` : A 25-point integration formula is used.
= `64` : A 64-point integration formula is used.

`EPS` (type according to `t`) Accuracy parameter (see **Accuracy**).

`X1,Y1` (type according to `t`) The coordinates of the vertices of Δ.

`X2,Y2`

`X3,Y3`

Method:

$\texttt{NSD} = \texttt{0}$:
An approximation I_0 to I is found by computing the NPT-point formula for the triangle Δ. The value of EPS has no influence on the result.
$\texttt{NSD} = \texttt{1}$:
After computing I_0, the triangle Δ is subdivided into two subtriangles Δ' and Δ'', the corresponding approximations I' and I'' are computed, and a test is made to see whether

$$\frac{|I_0 - (I' + I'')|}{1 + |I' + I''|} < \texttt{EPS}$$

If this test is satisfied, the routine terminates by setting the function value to I_0. If it fails, the process of subdivision and testing continues according to a tree structure. The routine terminates either because the test is passed successfully by all the subtriangles at some level, or because a maximum number of subdivisions is reached (see **Error Handling**).

Accuracy:

Unless there is severe cancellation of positive and negative values of $f(x, y)$ over Δ,the argument EPS may, if $\texttt{NCD} = \texttt{1}$, be considered as specifying a bound on the relative error of I in the case $|I| > 1$, and a bound on the absolute error in the case $|I| < 1$.

Restrictions:

"Mild" singularities are permitted if they coincide with the vertices of Δ. Any other singularity lying inside Δ or on its boundaries will most likely lead to too many subdivisions (see **Error Handling**), or cause a wrong result.

Error handling:

Error D105.1: $\texttt{NPT} \neq \texttt{7, 25, 64}$.
Error D105.2: The number of subdivisions has reached 35 without success.
In both cases, the function value is set equal to zero, and a message is written on Unit 6, unless subroutine MTLSET (N002) has been called.

References:

1. K.S. Kölbig, A Fortran program and some numerical test results for the integration over a triangle, CERN 64–32 (1964).

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