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Gaussian Quadrature for Multiple Integrals

Function subprogram packages `RGMLT` and `DGMLT` compute an approximate value of an n-dimensional integral of the form

$$I_n = \int_{a_n}^{b_n} dx_n \phi_n(x_n) \int_{a_{n-1}(x_n)}^{b_{n-1}(x_n)} dx_{n-1} \phi_{n-1}(x_{n-1}, x_n) \cdots$$

$$\phi_2(x_2, \ldots, x_n) \int_{a_1(x_2,\ldots,x_n)}^{b_1(x_2,\ldots,x_n)} dx_1 \phi_1(x_1, \ldots, x_n),$$

where $1 \le n \le 6$.

Each subprogram integrates over only one variable. The integral I_n is computed by means of a set of n nested calls to these subprograms.

On computers other than CDC or Cray, only the double-precision version `DGMLT` is available. On CDC and Cray computers, only the single-precision version `RGMLT` is available.

Structure:

`FUNCTION` subprograms
User Entry Names: `RGMLT1`, `RGMLT2`, `RGMLT3`, `RGMLT4`, `RGMLT5`, `RGMLT6`,
`DGMLT1`, `DGMLT2`, `DGMLT3`, `DGMLT4`, `DGMLT5`, `DGMLT6`
Files Referenced: `Unit 6`
External References: `MTLMTR` (N002), `ABEND` (Z035), user-supplied `SUBROUTINE` subprograms

Usage:

1. Let `k` be one of the integers `1`, `2`, . . . , `6`. Then, in any arithmetic expression,

   `RGMLTk(FSUBk,Ak,Bk,NIk,NGk,X)` or
   `DGMLTk(FSUBk,Ak,Bk,NIk,NGk,X)`

   has the approximate value of the integral with respect to x_k of the function specified below.

   `RGMLTk` is of type `REAL`, `DGMLTk` is of type `DOUBLE PRECISION`, and the arguments `Ak`, `Bk`, and `X` have the same type as the function name. `NIk` and `NGk` are of type `INTEGER`.

   `FSUBk` Name of a user-supplied `SUBROUTINE` subprogram, declared `EXTERNAL` in the calling program.
   `Ak,Bk` End points of the integration interval for variable x_k.
   `NIk` Number of equal subintervals into which the interval `(Ak,Bk)` is divided.
   `NGk` Number of Gaussian quadrature points to be used in each of the `NIk` subintervals. (If `NGk` has any value other than `6` or `8`, the value `6` is assumed).
   `X` One-dimensional array of dimension $\ge n$.

2. The subroutine FSUBk should be of the form

```
SUBROUTINE FSUBk (M,Uk,Fk,X)
```

where Uk, Fk and X are of type REAL for RGMLTk and of type DOUBLE PRECISION for DGMLTk, and where M is of type INTEGER.

M An integer ($\leq$ 64), whose value is set by RGMLTk (DGMLTk).

Uk One-dimensional array with declared dimension (*), whose contents is set by RGMLTk (DGMLTk).

Fk One-dimensional array with declared dimension (*), whose contents must be set by FSUBk as described below.

X One-dimensional array which must be the same as the array X appearing as an actual argument in all calls to RGMLT1, RGMLT2,. . . (DGMLT1, DGMLT2, . . .).

The subprogram RGMLTk (DGMLTk) which calls subroutine FSUBk sets the value of M and places in array Uk a set of M values of the variable x_k. Then, if $f_k(x_k, \ldots, x_n)$ denotes the function which is to be integrated with respect to x_k, it is the job of subroutine FSUBk to set X(k) to the appropriate value of x_k, to compute f_k for each of these values of x_k (taking the remaining variables $x_{k+1}, \ldots, x_n$ from X(k+1),. . .,X(n) respectively) and place the results in array Fk. (See **Examples**).

Method:

Integration with respect to each variable is performed by applying the 6- or 8-point Gaussian quadrature formula to each of the equal sub-intervals.

Notes:

1. The time needed to compute an n-dimensional integral by means of these subprograms is approximately
$$(\mathtt{NG1} * \mathtt{NG2} * \cdots * \mathtt{NGn}) * (\mathtt{NI1} * \mathtt{NI2} * \cdots * \mathtt{NIn}).$$
This should be kept in mind when choosing the values of NIk.

2. The accuracy of a particular calculation can be estimated by repeating the integration with different values of NGk (e.g., 8 instead of 6) or by changing NIk, the latter usually being less economical.

Error handling:

Error D110.1: NIk $\leq$ 0. A message is written on Unit 6, unless subroutine MTLSET (N002) has been called. Execution is halted on a STOP instruction.

Examples:

To calculate (in type REAL) the integral

$$Q_2 = \int_0^1 dx_2 \sqrt{x_2} e^{x_2} \int_0^{\sqrt{x_2}} dx_1 x_1 \sqrt{x_1^2 + x_2} = \frac{1}{3}(2\sqrt{2} - 1)(e - 2)$$

using 6-point Gaussian quadrature over each of $n_2 = 3$, $n_1 = 4$ subdivisions of the corresponding interval. In the main program, write for instance

```
      ...
      EXTERNAL FSUB2
      DIMENSION X(2)
      Q2=RGMLT2(FSUB2,0.,1.,3,6,X)
      ...
```

For the SUBROUTINE subprograms FSUB2, FSUB1 write for instance

```
      SUBROUTINE FSUB2(M,U2,F2,X)
      EXTERNAL FSUB1
      DIMENSION U2(*),F2(*),X(2)
      DO 1 L = 1,M
      X(2)=U2(L)
      R=SQRT(X(2))
    1 F2(L)=R*EXP(X(2))*RGMLT1(FSUB1,0.,R,4,6,X)
      RETURN
      END

      SUBROUTINE FSUB1(M,U1,F1,X)
      DIMENSION U1(*),F1(*),X(2)
      DO 1 L = 1,M
      X(1)=U1(L)
    1 F1(L)=X(1)*SQRT(X(1)**2+X(2))
      RETURN
      END
```

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