Author(s) : G.A. Erskine
Submitter :
Language : Fortran

Library: MATHLIB
Submitted: 01.09.1983
Revised: 01.03.1994

First-order Differential Equations (Runge–Kutta)

Subroutine subprograms RRKSTP and DRKSTP advance the solution of the system of $n \geq 1$ simultaneous first-order differential equations

$$\frac{dy_i}{dx} = f_i(x, y_1, \ldots, y_n), \qquad (i = 1, 2, \ldots, n)$$

by a single step of length h in the independent variable x.
On CDC and Cray computers, the double-precision version DRKSTP is not available.

Structure:

SUBROUTINE subprograms
User Entry Names : RKSTP, DRKSTP
Obsolete User Entry Names : RKSTP ≡ RRKSTP
Files Referenced : Unit 6
External References: user-supplied SUBROUTINE subprogram

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

```
CALL tRKSTP(N,H,X,Y,SUB,W)
```

N	(INTEGER) Number n of equations.
H	(type according to t) The step-length h.
X	(type according to t) On entry, X must be equal to the initial value of the independent variable x. On exit, X is equal to $x + h$.
Y	(type according to t) One-dimensional array of length $\geq$ N. On entry, Y(i), (i = 1, . . . , N), must contain $y_i(x)$. On exit, Y(i), (i = 1, . . . , N), contains approximate values $y_i(x + h)$.
SUB	Name of a user-supplied SUBROUTINE subprogram, declared EXTERNAL in the calling program.
W	(type according to t) Array containing at least 3*N elements required as working-space.

The user-supplied subroutine SUB should be of the form

```
SUBROUTINE SUB(X,Y,F)
```

where the variable X and the one-dimensional arrays Y(*) and F(*) are of type t. This subroutine must set

$$\mathtt{F(I)} = f_{\mathtt{I}}(\mathtt{X}, \mathtt{Y(1)}, \ldots, \mathtt{Y(N)}) \qquad (\mathtt{I} = 1, 2, \ldots, \mathtt{N}).$$

Method:

Using boldface quantities to denote vectors of length n, the computational sequence is as follows:

$$\begin{aligned}
\mathbf{k}_1 &= h\mathbf{f}(x, \mathbf{y}(x)), \\
\mathbf{k}_2 &= h\mathbf{f}(x + \tfrac{1}{2}h, \mathbf{y}(x) + \tfrac{1}{2}\mathbf{k}_1), \\
\mathbf{k}_3 &= h\mathbf{f}(x + \tfrac{1}{2}h, \mathbf{y}(x) + \tfrac{1}{2}\mathbf{k}_2), \\
\mathbf{k}_4 &= h\mathbf{f}(x + h, \mathbf{y}(x) + \mathbf{k}_3); \qquad \mathbf{y}(x + h) = \mathbf{y}(x) + \tfrac{1}{6}(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4)
\end{aligned}$$

The error per step is proportional to h^5.

Error handling:

`N` $<$ `1` acts as do nothing.

References:

1. F.B. Hildebrand, Introduction to numerical analysis, (McGraw-Hill, New–York 1956) Sect. 6.16.

•