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Library: MATHLIB
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Submitted: 07.06.1992
Language : Fortran
Revised: 01.12.1994

Second-order Differential Equations (Runge–Kutta–Nyström)

Subroutine subprograms RRKNYS and DRKNYS advance the solution of the system of $n \geq 1$ simultaneous second-order differential equations

$$\frac{d^2y_i}{dx^2} = f_i(x, y_1, \ldots, y_n, y_1', \ldots, y_n'), \qquad (i = 1, 2, \ldots, n)$$

where $y_i' = dy_i/dx$, by a single step of length h in the independent variable x.

On computers other than CDC or Cray, only the double-precision version DRKNYS is available. On CDC and Cray computers, only the single-precision version RRKNYS is available.

Structure:

SUBROUTINE subprograms
User Entry Names : RRKNYS, DRKNYS
Obsolete User Entry Names: RKNYS $\equiv$ RRKNYS
External References: User-supplied SUBROUTINE subprogram

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

```
CALL tRKNYS(N,H,X,Y,YP,SUB,W)
```

N (INTEGER) Number n of equations.

H (type according to t) The step-length h.

X On entry, X must be equal to the initial value of the independent variable x. On exit, X is equal to $x + h$.

Y (type according to t) One-dimensional array of length $\geq$ N. On entry, Y(i), (i = 1, ... , N), must contain $y_i(x)$. On exit, Y(i), (i = 1, ... , N), contains approximate values $y_i(x + h)$.

YP (type according to t) One-dimensional array of length $\geq$ N. On entry, YP(i), (i = 1, ... , N), must contain $y_i'(x)$. On exit, YP(i), (i = 1, ... , N), contains approximate values $y_i'(x + h)$.

SUB Name of a user-supplied SUBROUTINE subprogram, declared EXTERNAL in the calling program.

W (type according to t) Array containing at least 6*N elements required as working-space.

The user-supplied subroutine SUB should be of the form

```
SUBROUTINE SUB(X,Y,YP,F)
```

where the variable X and the one-dimensional arrays Y(*), YP(*) and F(*) are of type t. This subroutine must set

$$\mathtt{F(I)} = f_{\mathtt{I}}(\mathtt{X}, \mathtt{Y(1)}, \ldots, \mathtt{Y(N)}, \mathtt{YP(1)}, \ldots, \mathtt{YP(N)}) \qquad (\mathtt{I} = 1, 2, \ldots, \mathtt{N}).$$

Method:

Using boldface quantities to denote vectors of length n, the computational sequence is as follows:

$$
\begin{aligned}
\mathbf{k}_1 &= \frac{1}{2}h^2\,\mathbf{f}(x, \mathbf{y}(x), \mathbf{y}'(x)) \\
\mathbf{k}_2 &= \frac{1}{2}h^2\,\mathbf{f}(x+\frac{1}{2}h, \mathbf{y}(x)+\frac{1}{2}h\mathbf{y}'(x)+\frac{1}{4}\mathbf{k}_1, \mathbf{y}'(x)+\frac{1}{h}\mathbf{k}_1) \\
\mathbf{k}_3 &= \frac{1}{2}h^2\,\mathbf{f}(x+\frac{1}{2}h, \mathbf{y}(x)+\frac{1}{2}h\mathbf{y}'(x)+\frac{1}{4}\mathbf{k}_1, \mathbf{y}'(x)+\frac{1}{h}\mathbf{k}_2) \\
\mathbf{k}_4 &= \frac{1}{2}h^2\,\mathbf{f}(x+h, \mathbf{y}(x)+h\mathbf{y}'(x)+\mathbf{k}_3, \mathbf{y}'(x)+\frac{2}{h}\mathbf{k}_3) \\
\mathbf{y}(x+h) &= \mathbf{y}(x)+h\mathbf{y}'(x)+\frac{1}{3}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \\
\mathbf{y}'(x+h) &= \mathbf{y}'(x)+\frac{1}{3h}(\mathbf{k}_1+2\mathbf{k}_2+2\mathbf{k}_3+\mathbf{k}_4)
\end{aligned}
$$

The error per step is proportional to h^5.

Error handling:

For $\texttt{N} \le \texttt{0}$ or $\texttt{H} = \texttt{0}$, control is returned to the calling program without any change in `Y` or `YP`.

References:

1. L. Collatz, The numerical treatment of differential equations, (Springer-Verlag Berlin 1960) 537

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