Author(s) : C. Letertre
Submitter : B. Schorr
Language : Fortran

Library: KERNLIB
Submitted: 17.05.1971
Revised: 27.11.1984

Multidimensional Linear Interpolation

Function subprogram `FINT` uses repeated linear interpolation to evaluate a function $f(x_1, x_2, \ldots, x_n)$ of n variables which has been tabulated at the nodes of an n-dimensional rectangular grid. It is not necessary that the table arguments corresponding to any coordinate x_i be equally spaced.

Structure:

`FUNCTION` subprogram
User Entry Names: `FINT`
Files Refernced: Printer
External References: `KERMTR` (N001), `ABEND` (Z035)

Usage:

In any arithmetic expression,

```
FINT(N,X,NA,A,F)
```

has an approximate value of $f(\mathtt{X}_1, \mathtt{X}_2, \ldots, \mathtt{X}_\mathtt{n})$.

`N` (`INTEGER`) Number of coordinates n required to define the function f.

`X` (`REAL`) One-dimensional array. `X(j)`, $(j = 1, 2, \ldots, \mathtt{N})$, must contain the coordinates of the point at which the interpolation is to be performed.

`NA` (`INTEGER`) One-dimensional array. For $j = 1, 2, \ldots, \mathtt{N}$, `NA(j)` must be equal to the number of numerical values of variable x_j which are stored in array `A`.

`A` (`REAL`) One-dimensional array of length not less than the sum of `NA(1)`, ..., `NA(N)`. The first `NA(1)` elements of `A` must contain numerical values $a_{11}, a_{12}, \ldots$ of the first variable x_1 in strictly increasing order, the next `NA(2)` elements of `A` must contain numerical values $a_{21}, a_{22}, \ldots$ of the second variable x_2 in strictly increasing order, and so on.

`F` (`REAL`) Multidimensional array with dimensions `NA(1), NA(2), ...,NA(N)`, containing values of the function f at the nodes of the rectangular grid defined by `A`:
$\mathtt{F(i,j,\ldots,m)} = f(a_{1\mathtt{i}}, a_{2\mathtt{j}}, \ldots, a_{n\mathtt{m}})$, $(i = 1, 2, \ldots, \mathtt{NA(1)}, \ldots; m = 1, 2, \ldots, \mathtt{NA(N)})$.

If any coordinate x_i of the given point X lies outside the range of the corresponding table arguments, the interpolation for this coordinate is replaced by an extrapolation based on the two nearest table arguments, with consequent loss of accuracy.

Method:

Repeated linear interpolation with respect to variables $x_1, x_2, \ldots$ within the grid cell which contains the given point X. For $n = 2$, with (x_1, x_2) replaced by (x, y) for clarity, the procedure is equivalent to the following:

Let $a_1, a_2, \ldots$ be the tabulated values of x. Let $b_1, b_2, \ldots$ be the tabulated values of y.
Let i and j be the subscripts for which $a_i \le x < a_{i+1}, b_j \le y < b_{j+1}$.

Then compute:

$$
\begin{aligned}
t &= (x-a)/(a_{i+1}-a), \\
g_j &= (1-t)f(a_i,b_j)+tf(a_{i+1},b_j), \\
g_{j+1} &= (1-t)f(a_i,b_{j+1}+tf(a_{i+1},b_{j+1}), \\
u &= (y-b)/(b_{j+1}-b), \\
f_{appr} &= (1-u)g_j+ug_{j+1}
\end{aligned}
$$

Restrictions:

1. $\mathtt{1} \leq \mathtt{N} \leq \mathtt{5}$. `FINT` is set equal to zero if `N` is not in this range.
2. $\mathtt{NA(j)} \geq \mathtt{1}$, $(\mathtt{j} = \mathtt{1}, \mathtt{2}, \ldots, \mathtt{N})$.
3. The table arguments for each variable must be in strictly increasing order.

There is no test for conditions 2 or 3.

Error handling:

`E104.1`: $\mathtt{N} < \mathtt{1}$ or $\mathtt{N} > \mathtt{5}$. `FINT` is set equal to zero, and a message is printed unless subroutine `KERSET` `(N001)` has been called.

Examples:

Given a function of two variables $g(x,y)$ defined by a `FUNCTION` subprogram `G`, to construct a table of values of $f_{km} = g(\sqrt{k}, \log m)$ for $k = 1, 2, \ldots, 10$, $m = 1, 2, \ldots, 15$, and to interpolate in this table to set `GINT` equal to an approximate value of $g(1.7, 2.9)$. The program is written in a form which allows generalization to functions of more than two variables.

```
      PARAMETER (NA1=10,NA2=15)
      DIMENSION X(2),NA(2),A(NA1+NA2),F(NA1,NA2)
      DATA NA/NA1,NA2/
C     STORE ARGUMENT ARRAY
      K1=0
      K2=K1+NA1
      DO 1 J = 1,MAX(NA1,NA2)
        IF (J .LE. NA1) A(J+K1)=SQRT(FLOAT(J))
        IF (J .LE. NA2) A(J+K2)=LOG(FLOAT(J))
    1 CONTINUE
C     STORE FUNCTION ARRAY
      DO 3 J1 = 1,NA1
        DO 2 J2 = 1,NA2
          F(J1,J2)=G(A(J1+K1),A(J2+K2))
    2   CONTINUE
    3 CONTINUE
C     INTERPOLATE IN TABLE
      X(1)=1.7
      X(2)=2.9
      GINT=FINT(2,X,NA,A,F)
      ...
```

•