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Polynomial Splines / Normalized B-Splines

`NORBAS` (`NOR`malized `BA`sis `S`plines) is a portable collection of subprograms for various calculations with polynomial splines in one dimension (**1D**) and in two dimensions (**2D**). The polynomial splines are represented as linear combinations of normalized basis splines (B-splines).

On computers other than CDC or Cray, only the double-precision versions `DSPKN1`, etc. are available. On CDC and Cray computers, only the single-precision versions `RSPKN1`, etc. are available.

The following outline provides the background material and the notations needed for describing the subprograms and their parameters. For further information about splines and their applications see **References**, in particular Ref. 7.

Case (1D):

k Degree (order -1) of the B-spline $(0 \le k \le 25)$.

m Number of spline-knots $(m \ge 2\,k+2)$.

i Index of the B-spline $(1 \le i \le m-k-1)$.

τ Set of m spline-knots $\tau = \{t_1, t_2, \ldots, t_m\}$, in non-decreasing order, with multiplicity $\le k+1$ (i.e. no more than $k+1$ knots coincide).

$[a,b]$ Interval, defined by $a = t_{k+1}$, $b = t_{m-k}$.

$B_i(x)$ Normalized B-spline of degree k over τ with index i. The value of $B_i(x)$ is identically zero outside the interval $t_i \le x \le t_{i+k+1}$, and the normalization of $B_i(x)$ is such that

$$\int_{-\infty}^{+\infty} B_i(x)\,dx \;=\; \frac{t_{i+k+1}-t_i}{k+1} \qquad (i = 1, \ldots, m-k-1).$$

$s(x)$ Polynomial spline at $x \in [a,b]$ in B-spline representation

$$y \;=\; s(x) \;=\; \sum_{i=1}^{m-k-1} c_i\,B_i(x)\,.$$

Spline interpolation to a data set:

Given a data set $\{x_l, y_l\}_{l=1,\ldots,n}$; determine coefficients $\{c_i\}_{i=1,\ldots,n}$ of a polynomial interpolation spline $y = s(x)$ in B-spline representation with degree k over a set τ of $m = n+k+1$ knots, such that the following relations (interpolation conditions) hold:

$$s(x_l) \;=\; y_l \qquad (l = 1, \ldots, n).$$

The existence of a solution of this interpolation problem depends on an appropriate choice of the spline-knots (Ref. 7, Theorem XIII.1 (Schoenberg-Whitney)).

Least squares spline approximation to a data set:

Given a data set $\{x_l, y_l\}_{l=1,\ldots,n}$; determine coefficients $\{c_i\}_{i=1,\ldots,m-k-1}$ of a polynomial approximation spline $y = s(x)$ in B-spline representation with degree k over a set τ of $m \le n+k+1$ knots, such that following least squares problem is solved:

$$\phi(c_1, \ldots, c_{m-k-1}) \;=\; \sum_{l=1}^{n} \left(y_l - s(x_l)\right)^2 \;=\; \mathtt{min!}$$

Variation diminishing spline approximation to a function (Schoenberg):
For a given function $y = f(x)$ on $[a, b]$ this spline approximation is defined by $y = s(x)$, with (Ref. 7, p. 158-162)

$$c_i \;=\; f(x_i); \qquad x_i \;=\; (t_{i+1} + \cdots + t_{i+k})/k \quad (i = 1, \ldots, m - k - 1;\; k \geq 1).$$

Case (2D):

kx	Degree of one-dimensional B-splines in x-direction $(0 \leq kx \leq 25)$.
ky	Degree of one-dimensional B-splines in y-direction $(0 \leq ky \leq 25)$.
mx	Number of spline-knots in x-direction $(mx \geq 2\,kx + 2)$.
my	Number of spline-knots in y-direction $(my \geq 2\,ky + 2)$.
i	Index of B-spline $(1 \leq i \leq mx - kx - 1)$ in x-direction.
j	Index of B-spline $(1 \leq j \leq my - ky - 1)$ in y-direction.
τ_x	Set of mx spline-knots $\tau_x = \{t_{x,1}, t_{x,2}, \ldots, t_{x,mx}\}$ in x-direction, in non-decreasing order, with multiplicity $\leq kx + 1$ (i.e. no more than $kx + 1$ knots coincide).
τ_y	Set of my spline-knots $\tau_y = \{t_{y,1}, t_{y,2}, \ldots, t_{y,my}\}$ in y-direction, in non-decreasing order, with multiplicity $\leq ky + 1$ (i.e. no more than $ky + 1$ knots coincide).
$[a_x, b_x]$	Interval in x-direction, defined by $a_x = t_{x,kx+1}$, $b_x = t_{x,mx-kx}$.
$[a_y, b_y]$	Interval in y-direction, defined by $a_y = t_{y,ky+1}$, $b_y = t_{y,my-ky}$.
$B_i(x)$	B-spline of degree kx over τ_x with index i.
$B_j(y)$	B-spline of degree ky over τ_y with index j.
$B_{i,j}(x, y)$	Product $B_{i,j}(x, y) = B_i(x)\, B_j(y)$ of two one-dimensional B-splines.
$s(x, y)$	Two-dimensional polynomial spline at $(x, y) \in [a_x, b_x] \times [a_y, b_y]$ in B-spline representation

$$z \;=\; s(x, y) \;=\; \sum_{i=1}^{mx-kx-1} \; \sum_{j=1}^{my-ky-1} \; c_{i,j}\, B_{i,j}(x, y).$$

Spline interpolation to a data set:
Given a data set $\{x_{lx}, y_{ly}, z_{lx,ly}\}_{lx=1,\ldots,nx;ly=1,\ldots,ny}$; determine coefficients $\{c_{i,j}\}_{i=1,\ldots,nx;j=1,\ldots,ny}$ of a two-dimensional polynomial interpolation spline $z = s(x, y)$ in B-spline representation with degrees kx , ky over the sets τ_x of $mx = nx + kx + 1$ knots in x-direction and τ_y of $my = ny + ky + 1$ knots in y-direction, such that following relations (interpolation conditions) hold:

$$s(x_{lx}, y_{ly}) = z_{lx,ly} \qquad (lx = 1, \ldots, nx\,;\; ly = 1, \ldots, ny).$$

The existence of a solution of this interpolation problem depends on an appropriate choice of the spline-knots τ_x , τ_y in the two-dimensional interpolation area $[a_x, b_x] \times [a_y, b_y]$.
Least squares spline approximation to a data set:
Given a data set $\{x_{lx}, y_{ly}, z_{lx,ly}\}_{lx=1,\ldots,nx;ly=1,\ldots,ny}$; determine coefficients $\{c_{i,j}\}_{i=1,\ldots,nx;j=1,\ldots,ny}$ of a two-dimensional polynomial approximation spline $z = s(x, y)$ in B-spline representation with degrees kx, ky over the sets τ_x of $mx \leq nx + kx + 1$ knots in x-direction and τ_y of $my \leq ny + ky + 1$ knots in y-direction, such that following least squares problem is solved:

$$\phi(c_{1,1}, \ldots, c_{mx-kx-1,my-ky-1}) = \sum_{lx=1}^{nx} \sum_{ly=1}^{ny} \left(z_{lx,ly} - s(x_{lx}, y_{ly})\right)^2 \;=\; \texttt{min!}$$

Variation diminishing spline approximation to a function:
For a given function $z = f(x, y)$ on $[a_x, b_y] \times [a_y, b_y]$ this two-dimensional spline approximation is defined by $z = s(x, y)$ on $[a_x, b_x] \times [a_y, b_y]$, with

$$\begin{aligned} c_{i,j} = f(x_i, y_j); \qquad & x_i = (t_{x,i+1} + \cdots + t_{x,i+kx})/kx \quad (i = 1, \ldots, mx - kx - 1;\; kx \geq 1), \\ & y_j = (t_{y,j+1} + \cdots + t_{y,j+ky})/ky \quad (j = 1, \ldots, my - ky - 1;\; ky \geq 1). \end{aligned}$$

The package `NORBAS` contains `FUNCTION` and `SUBROUTINE` subprograms for solving the following problems.
To calculate:

(K) A set τ of m spline-knots in the interval $[a, b]$ for normalized B-splines $B_i(x)$ of degree k, use subprogram `tSPKN1` (**1D**). The knots are in non-decreasing order and determined in such a way that the first $k+1$ knots coincide with a, the last $k+1$ knots coincide with b, and the remaining $(m-2\,k-2)$ knots are equidistant in (a, b).
Two sets τ_x , τ_y of spline-knots in $[a_x, b_x]$ and $[a_y, b_y]$ for B-splines $B_{i,j}(x, y)$ of degrees kx and ky in x- and y-direction, use subprogram `tSPKN2` (**2D**). τ_x and τ_y, are calculated by the same formulae in x-, and y-direction, as in case (**1D**).

(B) The function $B_i(x)$,

$$\text{the } n\text{-th derivative} \quad \frac{d^n B_i(x)}{dx^n}, \qquad \text{or the integral} \quad \int_{-\infty}^{x} B_i(\xi)\, d\xi$$

of a normalized B-spline $B_i(x)$, with fixed degree k and index i over a set τ of spline-knots, use subprogram: `tSPNB1` (**1D**).
The function $B_{i,j}(x, y)$,

$$\text{the partial derivative} \quad \frac{\partial^{nx}\, \partial^{ny}\, B_{i,j}(x, y)}{\partial x^{nx}\, \partial y^{ny}}, \qquad \text{or the integral} \quad \int_{-\infty}^{x} \int_{-\infty}^{y} B_{i,j}(\xi, \eta)\, d\eta\, d\xi$$

of a two-dimensional B-spline $B_{i,j}(x, y)$, with fixed degrees kx, ky and indices i, j over the sets τ_x, τ_y of spline-knots, use subprogram `tSPNB2` (**2D**).

(P) The function $s(x)$,

$$\text{the } n\text{-th derivative} \quad \frac{d^n s(x)}{dx^n}, \qquad \text{or the integral} \quad \int_{-\infty}^{x} s(\xi)\, d\xi$$

of a polynomial spline $y = s(x)$ in B-spline representation with given coefficients c_i, use subprogram `tSPPS1` (**1D**).
The function $s(x, y)$,

$$\text{the partial derivative} \quad \frac{\partial^{nx}\, \partial^{ny}\, s(x, y)}{\partial x^{nx}\, \partial y^{ny}}, \qquad \text{or the integral} \quad \int_{-\infty}^{x} \int_{-\infty}^{y} s(\xi, \eta)\, d\eta\, d\xi$$

of a two-dimensional polynomial spline $z = s(x, y)$ in B-spline representation with given coefficients $c_{i,j}$, use subprogram `tSPPS2` (**2D**).

(I) The coefficients c_i of a one-dimensional polynomial interpolation spline $y = s(x)$ in B-spline representation to a user-supplied data set $\{x_l, y_l\}$, use subprogram `tSPIN1` (**1D**).
The coefficients $c_{i,j}$ of a two-dimensional polynomial interpolation spline $z = s(x, y)$ in B-spline representation to a user-supplied data set $\{x_{lx}, y_{ly}, z_{lx,ly}\}$, use subprogram `tSPIN2` (**2D**).

(A) The coefficients c_i of a one-dimensional polynomial least squares approximation spline $y = s(x)$ in B-spline representation to a user-supplied data set $\{x_l, y_l\}$, use subprogram `tSPAP1` (**1D**).
The coefficients $c_{i,j}$ of a two-dimensional polynomial least squares approximation spline $z = s(x, y)$ in B-spline representation to a user-supplied data set $\{x_{lx}, y_{ly}, z_{lx,ly}\}$, use subprogram `tSPAP2` (**2D**).

(V) The coefficients c_i of a one-dimensional polynomial variation diminishing spline approximation $y = s(x)$ in B-spline representation to a user-supplied function $y = f(x)$, use subprogram `tSPVD1` (**1D**).
The coefficients $c_{i,j}$ of a two-dimensional polynomial variation diminishing spline approximation $z = s(x, y)$ in B-spline representation to a user-supplied function $z = f(x, y)$, use subprogram `tSPVD2` (**2D**).

(D) From given coefficients c_i of a one-dimensional polynomial spline $y = s(x)$ in B-spline representation, the corresponding coefficients d_i of its n-th derivative $d^n s(x)/dx^n$ in B-spline representation, use subprogram `tSPCD1` (**1D**).
From given coefficients $c_{i,j}$ of a two-dimensional polynomial spline $z = s(x, y)$ in B-spline representation, the corresponding coefficients $d_{i,j}$ of its partial derivative $\partial^{nx}\, \partial^{ny}\, s(x, y)/\{\partial x^{nx}\, \partial y^{ny}\}$ in B-spline representation, use subprogram `tSPCD2` (**2D**).

Structure:

SUBROUTINE and FUNCTION subprograms
User Entry Names: RSPKN1, RSPKN2, RSPNB1, RSPNB2, RSPPS1, RSPPS2, RSPIN1, RSPIN2, RSPAP1, RSPAP2, RSPVD1, RSPVD2, RSPCD1, RSPCD2, DSPKN1, DSPKN2, DSPNB1, DSPNB2, DSPPS1, DSPPS2, DSPIN1, DSPIN2, DSPAP1, DSPAP2, DSPVD1, DSPVD2, DSPCD1, DSPCD2,
Internal Entry Names: RSPAS1, RSPAS2, RSPLKK, DSPAS1 DSPAS2, DSPLKK
Files Referenced: Unit 6
External References: RGBTRF (F001), RGBTRS (F001), RGESVD (F001), DGBTRF (F001), DGBTRS (F001), DGESVD (F001), RVSET (F002), RVSUM (F002), RVCPY (F002), RVMPY (F002), DVSET (F002), DVSUM (F002), DVCPY (F002), DVMPY (F002), RMCPY (F003), RMMPY (F003), DMCPY (F003), DMMPY (F003), MTLMTR (N002), ABEND (Z035).
User-supplied FUNCTION subprogram

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION):

(K) Knots

```
CALL tSPKN1(K,M,A,B,T,NERR)
CALL tSPKN2(KX,KY,MX,MY,AX,BX,AY,BY,TX,TY,NERR)
```

(B) Normalized B-splines

```
tSPNB1(K,M,I,NDER,X,T,NERR)
tSPNB2(KX,KY,MX,MY,I,J,NDERX,NDERY,X,Y,TX,TY,NERR)
```

(P) Polynomial spline

```
tSPPS1(K,M,NDER,X,T,C,NERR)
tSPPS2(KX,KY,MX,MY,NDERX,NDERY,X,Y,TX,TY,C,NDIMC,W,NERR)
```

(I) Spline interpolation

```
CALL tSPIN1(K,N,XI,YI,KNOT,T,C,W,IW,NERR)
CALL tSPIN2(KX,KY,NX,NY,XI,YI,ZI,NDIMZ,KNOT,TX,TY,C,NDIMC,W,IW,NERR)
```

(A) Least squares spline approximation

```
CALL tSPAP1(K,M,N,XI,YI,KNOT,T,C,W,NW,NERR)
CALL tSPAP2(KX,KY,MX,MY,NX,NY,XI,YI,ZI,NDIMZ,KNOT,TX,TY,C,NDIMC,W,NW,NERR)
```

(V) Variation diminishing spline approximation

```
CALL tSPVD1(F,K,M,T,C,NERR)
CALL tSPVD2(F,KX,KY,MX,MY,TX,TY,C,NDIMC,NERR)
```

(D) Coefficients of derivatives

```
CALL tSPCD1(K,M,NDER,T,C,D,NERR)
CALL tSPCD2(KX,KY,MX,MY,NDERX,NDERY,TX,TY,C,NDIMC,D,NERR)
```

Case (1D):

F Name of a user-upplied FUNCTION subprogram, declared EXTERNAL in the calling program. This subprogram must provide the value of the function $y = f(x)$ for variation diminishing spline approximation.

K (INTEGER) Degree of B-splines ($1 \leq$ K ≤ 25 for tSPVD1, $0 \leq$ K ≤ 25 otherwise).

M (INTEGER) Number of knots ($\geq 2 *$ K $+ 2$).

I (INTEGER) Index of B-splines ($1 \leq$ I $\leq$ M $-$ K $- 1$).

N (INTEGER) Number of data points $\{x_l, y_l\}$ ($\geq$ K $+ 1$ for spline interpolation (tSPIN1); $\geq$ M $-$ K $- 1$ for spline approximation (tSPAP1)).

NDER (INTEGER) Selects one out of three cases: (i) integral, (ii) function value, or (iii) derivative. ($-1 \leq$ NDER $\leq$ K for tSPNB1 and tSPPS1; $1 \leq$ NDER $\leq$ K for tSPCD1).

$= -1$: Calculation of the integral of $B_i(x)$ (tSPNB1), or the integral of $s(x)$ (tSPPS1).

$= 0$: Calculation of the function value $B_i(x)$ (tSPNB1), or the function value $s(x)$ (tSPPS1).

≥ 1 : Calculation of the NDER-th derivative of $B_i(x)$ (tSPNB1), or the NDER-th derivative of $s(x)$ (tSPPS1).

X (Type according to t) Independent variable x of polynomial spline $s(x)$ or B-spline $B_i(x)$.

XI (Type according to t) One-dimensional array of length $\geq$ N. On entry, XI(L) must contain the l-th data point x_l for spline interpolation (tSPIN1) or spline approximation (tSPAP1). The data points must be in ascending order.

YI (Type according to t) One-dimensional array of length $\geq$ N. On entry, YI(L) must contain the l-th data point y_l for spline interpolation (tSPIN1) or spline approximation (tSPAP1).

KNOT (INTEGER) Controls the mode of supplying the knots for spline interpolation or approximation.

$= 1, 2$: The knots are computed by the subprograms tSPIN1 and tSPAP1. At the left and right end-point of the interpolation (approximation) interval $[x_1, x_n]$ arise multiple knots. The remaining knots are either equidistant (KNOT $= 1$) or are computed by using the data points $\{x_l\}$ of interpolation (approximation) (KNOT $= 2$).

$\neq 1, 2$: The knots must be supplied by the user.

A,B (Type according to t) On entry, A and B must contain the left (right) end-point of the interval $[a, b]$ for the calculation of a set of spline knots (tSPKN1).

T (Type according to t) One-dimensional array of length $\geq$ M .
For tSPKN1 and for tSPINT1, tSPAP1 if KNOT $= 1, 2$: On exit, T(J) contains the j-th knot. In the other cases, on entry, T(J) must contain the j-th knot. The knots must be in non-decreasing order with multiplicity $\leq$ K $+ 1$.

C (Type according to t) One-dimensional array of length $\geq$ M $-$ K $- 1$.
For tSPPS1 and tSPCD1: On entry, C(I) must contain the i-th coefficient c_i of the polynomial spline $s(x)$ in B-spline representation.
For tSPIN1, tSPAP1 and tSPVD1: On exit, C(I) contains the i-th coefficient c_i of the polynomial interpolation or approximation spline $s(x)$ in B-spline representation.

D (Type according to t) One-dimensional array of length $\geq$ M $-$ K $- 1$.
On exit, D(I) contains the coefficient d_i of the NDER-th derivative of a polynomial spline $s(x)$ in B-spline representation.

W (Type according to t) One-dimensional array of length $\geq (3 *$ K $+ 1) *$ N (tSPIN1), and of length $\geq$ NW (tSPAP1); used as working space.

NW (INTEGER) Length of working array W, (NW $\geq$ N $* (n_0+5)+n_0*(n_0+1)$; $n_0=$ M $-$ K $- 1$).

IW (INTEGER) One-dimensional array of length $\geq$ N, used as working space.

NERR (INTEGER) Error indicator. On exit:

$= 0$: No error or warning detected.

$= 1$: At least one of the parameters I, K, M, N, NDER is not in range or $A \leq B$ is not true.

$= 2$: The subprograms tGEQPF, tORMQR, tTRTRS in the Linear Algebra package LAPACK (F001) were unable to solve the linear system of equations for calculating the coefficients of the spline interpolation to a given data set.

Case (2D):

F Name of a user-upplied FUNCTION subprogram, declared EXTERNAL in the calling program. This subprogram must provide the value of the function $z = f(x, y)$ for variation diminishing spline approximation.

KX,KY (INTEGER) Degree of one-dimensional B-splines in x- (y-)direction ($1 \leq KX \leq 25$, $1 \leq KY \leq 25$ for tSPVD2; $0 \leq KX \leq 25$, $0 \leq KY \leq 25$ otherwise).

MX,MY (INTEGER) Number of knots in x- (y-)direction ($MX \geq 2*KX+2$, $MY \geq 2*KY+2$).

I,J (INTEGER) Indices of B-splines ($1 \leq I \leq MX-KX-1$, $1 \leq J \leq MY-KY-1$).

NX,NY (INTEGER) Number of data points x_{lx} (y_{ly}) in x- (y-)direction ($NX \geq KX+1$, $NY \geq KY+1$ for spline interpolation tSPIN2; $NX \geq MX-KX-1$, $NY \geq MY-KY-1$ for spline approximation tSPAP2).

NDERX, NDERY (INTEGER) Selects one out of three cases: (i) integral, (ii) function value, or (iii) derivative. ($-1 \leq NDERX \leq KX$, $-1 \leq NDERY \leq KY$ for tSPNB2 and tSPPS2; $1 \leq NDERX \leq KX$, $1 \leq NDERY \leq KY$ for tSPCD2).

$= -1$: Calculation of the integral of $B_{i,j}(x, y)$ (tSPNB2), or the integral of $s(x, y)$ (tSPPS2).

$= 0$: Calculation of the function value $B_{i.j}(x, y)$ (tSPNB2), or the function value $s(x, y)$ (tSPPS2).

≥ 1 : Calculation of the NDERX-th partial derivative with respect to x and the NDERY-th partial derivative with respect to y of $B_{i,j}(x, y)$ (tSPNB2), or the calcultion of these derivatives of $s(x, y)$ (tSPPS2).

Note that in the first two cases $NDERX = NDERY = -1$, $NDERX = NDERY = 0$, respectively.

X,Y (Type according to t) Independent variables x, y of polynomial spline $s(x, y)$ or B-spline $B_{i,j}(x, y)$.

XI,YI (Type according to t) One-dimensional arrays of length $\geq$ NX and $\geq$ NY, respectively. On entry, XI(LX) and YI(LY) must contain the lx-th data point x_{lx} and the ly-th data point y_{ly} for spline interpolation (tSPIN2) or spline approximation (tSPAP2). The data points must be in ascending order.

ZI (Type according to t) Two-dimensional array of dimension (NDIMZ, $\geq$ NY). On entry, ZI(LX, LY) must contain the (lx, ly)-th data point $z_{lx,ly}$ for spline interpolation (tSPIN2) or spline approximation (tSPAP2).

NDIMZ (INTEGER) Declared first dimension of a two-dimensional array ZI in the calling program ($\geq$ NX).

KNOT (INTEGER) Controls the mode of supplying the knots for spline interpolation or approximation.

$= 1, 2$: The set of knots are computed by subprograms tSPIN2 and tSPAP2. At the left and right end-points of the interpolation (approximation) intervals $[x_1, x_{nx}]$, $[y_1, y_{ny}]$ arise multiple knots. The remaining knots are either equidistant ($KNOT = 1$) or are computed by using the data points $\{x_{lx}, y_{ly}\}$ of interpolation (approximation) ($KNOT = 2$).

$\neq 1, 2$: The knots must be supplied by the user.

AX,BX; AY,BY (Type according to t) On entry, AX, BX; AY, BY must contain the left (right) end-points of the intervals $[a_x, b_x]$; $[a_y, b_y]$ for the calculation of a set of spline knots in x- (y-)direction, respectively, by tSPKN2.

TX,TY (Type according to t) One-dimensional arrays of length $\geq$ MX and $\geq$ MY, repectively.
For tSPKN2 and for tSPIN2, tSPAP2 if KNOT $= 1, 2$: On exit, TX(J) and TY(J) contain the j-th knot in x- (y-)direction. In the other cases, on entry, TX(J) and TY(J) must contain the j-th knot in x- (y-)direction. The knots must be in non-decreasing order with multiplicity $\leq$ KX + 1 and $\leq$ KY + 1, respectively.

C (Type according to t) Two-dimensional array of dimension (NDIMC, $\geq$ MY $-$ KY $-$ 1).
For tSPPS2, tSPCD2: On entry, C(I,J) must contain the (i, j)-th coefficient $c_{i,j}$ of the polynomial spline $s(x, y)$ in B-spline representation.
For tSPIN2, tSPAP2, tSPVD2: On exit, C(I,J) contains the (i, j)-th coefficient $c_{i,j}$ of the polynomial interpolation or approximation spline $s(x, y)$ in B-spline representation.

NDIMC (INTEGER) Declared first dimension of a two-dimensional array C in the calling program ($\geq$ MX $-$ KX $-$ 1).

D (Type according to t) Two-dimensional array of dimension (NDIMC, $\geq$ MY $-$ KY $-$ 1).
On exit, D(I,J) contains the coefficient $d_{i,j}$ of the partial derivative of order nx, ny with respect to x and y of a polynomial spline $s(x, y)$ in B-spline representation.

W (Type according to t) One-dimensional array of length $\geq$ MY $-$ KY $-$ 1 (tSPPS2), $\geq$ (3 $*$ KX $*$ NY + 2) $*$ NX $*$ NY (tSPIN2), and of length $\geq$ NW (tSPAP2), used as working space.

NW (INTEGER) Length of a one-dimensional array W, used as working space ($\geq$ NX $*$ NY $*$ (n_0+6)+$n_0$$*$($n_0$+1); n_0= (MX $-$ KX $-$ 1) $*$ (MY $-$ KY $-$ 1)).

IW (INTEGER) One-dimensional array of length $\geq$ NX $*$ NY, used as working space.

NERR (INTEGER) Error indicator. On exit:

= 0 : No error or warning detected.

= 1 : At least one of the parameters I, J, KX, KY, MX, MY, NX, NY, NDERX, NDERY is not in range or at least one of the relations AX $\leq$ BX, AY $\leq$ BY is not true.

= 2 : The routines tGEQPF, tORMQR, tTRTRS in the Linear Algebra package LAPACK (F001) were unable to solve the linear system of equations for calculation coefficients of spline interpolation to a given data set.

Examples:

Calculate

1. The coefficients c_i of a polynomial interpolation spline $y = s(x)$ of degree $k = 2$ in B-spline representation to a given data set $\{(x_l, y_l)\}_{l=1,\dots,6}$;

2. The corresponding coefficients d_i of the first derivative $y' = \dfrac{ds(x)}{dx}$;

3. The values of $s(x)$, $\dfrac{ds(x)}{dx}$ and $\displaystyle\int_0^x s(\xi)d\xi$ for $x = 0(0.1)1$.

```
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION XI(6),YI(6),T(9),C(6),D(6),W(42),IW(6)
DATA K,N,NDER,KNOT / 2,6,1,1 /
DATA XI / 0D0,0.20D0,0.40D0,0.60D0,0.80D0,1.00D0 /
DATA YI / 1D0,0.66D0,0.47D0,0.38D0,0.35D0,0.34D0 /
M=N+K+1

CALL DSPIN1(K,N,XI,YI,KNOT,T,C,W,IW,NERR)
CALL DSPCD1(K,M,NDER,T,C,D,NERR)
```

```
      WRITE(6,1000) K,N,(T(I),I=1,M)
      DO 20 J=0,10
      X=J/1D1

      SPL0=DSPPS1(K,M, 0,X,T,C,NERR)
      SPL1=DSPPS1(K,M, 1,X,T,D,NERR)
      SPLI=DSPPS1(K,M,-1,X,T,C,NERR)

   20 WRITE(6,1010) J,X,SPL0,SPL1,SPLI
 1000 FORMAT(...)
 1010 FORMAT(...)
      END
```

```
 DEGREE OF POLYNOMIAL SPLINE:  2          NUMBER OF DATA POINTS:  6
 KNOTS T :        0.00  0.00  0.00  0.25  0.50  0.75  1.00  1.00  1.00

     J        X         S(X)          DS(X)         IN(X)
     0      0.00      1.000000     -2.119921      0.000000
     1      0.10      0.809004     -1.700000      0.090100
     2      0.20      0.660000     -1.280079      0.163201
     3      0.30      0.550992     -0.940017      0.223467
     4      0.40      0.470000     -0.679816      0.274299
     5      0.50      0.415028     -0.419615      0.318334
     6      0.60      0.380000     -0.280953      0.357970
     7      0.70      0.358838     -0.142290      0.394796
     8      0.80      0.350000     -0.065306      0.430174
     9      0.90      0.344235     -0.050000      0.464873
    10      1.00      0.340000     -0.034694      0.499072
```

Error handling:

Error E210.1: K|KX,KY not in range,
Error E210.2: M|MX,MY not in range,
Error E210.3: I|I,J not in range,
Error E210.4: N|NX,NY not in range,
Error E210.5: NDER|NDERX,NDERY not in range,
Error E210.6: A,B|AX,BX;AY,BY inconsistent,
Error E210.7: NDERX|NDERY inconsistent.

In all cases, NERR is set = 1 (see above). A message is written on Unit 6, unless subroutine MTLSET (N002) has been called.

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