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Solution of Overdetermined Linear System in the Chebychev Norm

Subroutine subprograms `RCHEBN` and `DCHEBN` find the Chebyshev or minimax solution to a set of overdetermined linear equations $\mathbf{A}\mathbf{x} = \mathbf{b}$, i.e. the vector **x** which minimizes

$$c = \max_{1 \le i \le m} c_i = \max_{1 \le i \le m} \left| b_i - \sum_{j=1}^{n} a_{ij} x_j \right| .$$

On computers other than CDC or Cray, only the double-precision version `DCHEBN` is available. On CDC and Cray computers, only the single-precision version `RCHEBN` is available.

Structure:

`SUBROUTINE` subprograms
User Entry Names: `RCHEBN`, `DCHEBN`
External References: `RVSCA` (F002), `RVSCL` (F002), `RVSCS` (F002), `RVSET` (F002), `RVXCH` (F002), `DVSCA` (F002), `DVSCL` (F002), `DVSCS` (F002), `DVSET` (F002), `DVXCH` (F002)

Usage:

For `t` = `R` (type `REAL`), `t` = `D` (type `DOUBLE PRECISION`),

```
CALL tCHEBN(M,N,A,MDIM,B,TOL,RELERR,X,RESMAX,IRANK,ITER,ICODE)
```

`M` (`INTEGER`) Number m of equations.

`N` (`INTEGER`) Number n $(\le m)$ of unknowns.

`A` (type according to `t`) Two-dimensional array of dimension `(MDIM,d)`, where `d` $\ge n+3$. On entry, `A(I,J)` must contain the coefficients a_{ij} $(i = 1, \ldots, m;\ j = 1, \ldots, n)$ of matrix **A**. The contents of `A` is destroyed during execution.

`MDIM` (`INTEGER`) Declared first dimension of array `A`, where `MDIM` $\ge m+1$.

`B` (type according to `t`) One-dimensional array of length $\ge m+1$. On entry, the first m elements of `B` must contain the vector **b**. On exit, these elements contain the residuals c_i.

`TOL` Tolerance parameter which should be set to a value somewhat greater than the machine precision.

`RELERR` (type according to `t`) On entry, `RELERR` should be set to zero if the true minimax solution is required. (For `RELERR` non-zero see **Notes**).

`X` (type according to `t`) One-dimensional array of length $\ge n+3$. On exit, the first n elements of `X` contain the solution vector **x**.

`RESMAX` (type according to `t`) On exit, `RESMAX` contains the value c of the maximum residual.

`IRANK` (`INTEGER`) On exit, `IRANK` contains an estimate of the rank of the matrix `A`. (This estimate may depend on `TOL`).

`ITER` (`INTEGER`) On exit, `ITER` contains the number of simplex iterations performed.

`ICODE` (`INTEGER`) On exit, `ICODE` contains one of the following:
= `0` : Solution **x** is not unique,
= `1` : Solution **x** is unique,
= `2` : Calculation terminated prematurely because of rounding error.

Method:

Modified simplex method of linear programming applied to the dual of the stated minimax problem.

Notes:

1. If RELERR on entry contains a non-zero positive value r, RELERR on exit contains a value $r' < r$, and the computed solution $\mathbf{x}'$ in X and the maximum residual c' in RESMAX are such that $(c' - c)/c < r'$, where c is the maximum residual corresponding to the true minimax solution $\mathbf{x}$. By setting RELERR non-zero (e.g. RELERR = 0.1), the number of simplex iterations is usually reduced.
2. If RESMAX is within one or two orders of magnitude of TOL, the computed residuals in B on exit may contain few significant digits, and may have been set to zero if RESMAX $<$ TOL.

Source:

The subprograms are based on a Fortran algorithm given in Ref. 1.

References:

1. I. Barrodale and C. Phillips, Algorithm 495: Solution of an overdetermined system of linear equations in the Chebyshev norm, ACM Trans. Math. Software **1** (1975) 264–270.

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