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Chebyshev Series Coefficients of a Function

Subroutine subprograms `RCHECF`, `DCHECF` and `QCHECF` calculate coefficients for a finite sum of Chebyshev polynomials approximating a function $f(x)$ over an interval $a \le x \le b$ to accuracy ε. It returns an integer n and coefficients $c_0, c_1, \ldots, c_n$ such that the sum

$$f^*(x) = \sum_{j=0}^{n} c_j T_j(t) \qquad (1)$$

where $t = (2x - a - b)/(b - a)$ and $T_j(t)$ is the Chebyshev polynomial of degree j, satisfies for $a \le x \le b$ the relation

$$|f^*(x) - f(x)| < \varepsilon. \qquad (2)$$

Subsequent evaluation of the approximation (1) can be done by calling `CHSUM` (E407) with the appropriate value of its argument `MODE`.

On computers other than CDC and Cray, only the double- and quadruple-precision versions `DCHECF` and `QCHECF` are available. On CDC and Cray computers, only the single- and double-precision versions `RCHECF` and `DCHECF` are available.

Structure:

`SUBROUTINE` subprogram
User Entry Names: `RCHECF`, `DCHECF`, `QCHECF`
Obsolete User Entry Names: `CHECF` $\equiv$ `RCHECF`
Files Referenced: `Unit 6`
External References: `MTLMTR` (N002), `ABEND` (Z035), user-supplied `FUNCTION` subprogram

Usage:

For `t` = `R` (type `REAL`), `t` = `D` (type `DOUBLE PRECISION`), `t` = `Q` (type `REAL*16`),

```
CALL tCHECF(F,A,B,EPS,C,N,DELTA)
```

`F` (type according to `t`) Name of a user-supplied `FUNCTION` subprogram, declared `EXTERNAL` in the calling program.

`A,B` (type according to `t`) End-points a, b of the approximation interval.

`EPS` (type according to `t`) Requested accuracy.

`C` (type according to `t`) One-dimensional array with dimension `(0:d)`, `d` $\ge$ `128`. On exit, `C(j)` $= c_j$, ($j = 0, 1, \ldots,$ `N`).

`N` (`INTEGER`) On exit, `N` is equal to the subscript of the last computed coefficient.

`DELTA` (type according to `t`) On exit, `DELTA` is such that the relation $|f^*(x) - f(x)| <$ `DELTA` is almost certainly true for $x \in [a, b]$. (See Error Handling.)

Method:

The interval $[a, b]$ is subdivided successively into sets of subintervals of length $2^{-k}(b-a)$, $(k = 0, 1, 2 \ldots)$. After each subdivision the orthogonality properties of the Chebyshev polynomials with respect to summation over equally-spaced points are used to compute two sets of approximate values of the coefficients c_j: one set computed using the end-points of the subintervals, and one set using the mid-points. The mean of these two values is taken as the best estimate of the c_j, which are then tested to see (a) whether certain rate-of-convergence criteria are satisfied, (b) whether there is some n for which the sum for $j > n$ of the available c_j is less than ε. If both conditions are satisfied the subroutine terminates.

Error handling:

Error `E406.1`: If the requested accuracy cannot be obtained with 65 coefficients (i.e., $\mathtt{N} = 64$) a message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called. In this case, values of f^* computed from (1) with $\mathtt{N} = 64$ should still be in error by less than `DELTA`.

Notes:

1. This subroutine is intended for use with functions $f(x)$ which can be computed to full machine accuracy, and which are sufficiently smooth to ensure fairly rapid decrease of the c_j with increasing j. Functions defined by experimental data can usually be approximated better by least-squares methods, using ordinary polynomials.
2. Note that some authors use a different definition for the constant term in (1), i.e. $c_0/2$ instead of c_0. Here, the definition of Ref. 1 is used.

References:

1. Y.L. Luke, Mathematical functions and their approximations, (Academic Press, New York 1975)

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