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Linear Homogeneous Inequalities

Subroutine subprograms RLHOIN and DLHOIN find the basis $\mathbf{v}_j$, $(j = 1, 2, \ldots, J)$, of the convex polyhedral cone defining the solution of a system of homogeneous linear inequalities $\mathbf{Ax} \geq 0$. $\mathbf{A} = a_{mn}$ is a given $M \times N$ matrix, $M \geq N$, and rank$(\mathbf{A}) = N$. $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is a column vector. Any solution **x** of $\mathbf{Ax} \geq 0$ can be expressed as

$$\mathbf{x} = \sum_{j=1}^{J} \lambda_j \mathbf{v}_j.$$

where all $\lambda_j \geq 0$. The number J of vectors $\mathbf{v}_j$ depends on the matrix **A** in an unknown way, except when $M = N$, where $J = N$.

On CDC and Cray computers, the double-precision version DLHOIN is not available.

Structure:

SUBROUTINE subprogram
User Entry Names: RLHOIN, DLHOIN
Obsolete User Entry Names: LIHOIN ≡ RLHOIN
Files Referenced: Unit 6
External References: RVCPY (F002), RVMPY (F002), RVSCL (F002),
DVCPY (F002), DVMPY (F002), DVSCL (F002),
RMCPY (F003), RMSET (F003), DMCPY (F003), DMSET (F003),
RINV (F010), DINV (F010), MTLMTR (N002), ABEND (Z035)

Usage:

For t = R (type REAL), t = D (type DOUBLE PRECISION),

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CALL tLHOIN(A,MA,M,N,MAXV,V,NV,JVEC,EPS,IOUT,W,IW)
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A (type according to t) Two-dimensional array, dimensioned (MA, ≥ N), whose rows contain the coefficients of the inequalities, arranged in such a way that the upper left N × N corner has a non-vanishing determinant. Usually it is advisable to normalise the rows of A to unity before calling this subprogram.

MA (INTEGER) First dimension parameter of A.

M (INTEGER) Number M of inequalities.

N (INTEGER) Number N of variables.

MAXV (INTEGER) Maximum number of basis vectors which may occur at any intermediate step, to be chosen sufficiently large and in any case ≥ N.

V (type according to t) Two-dimensional array, dimensioned (NV, ≥ MAXV), whose columns contain, on return, the basis vectors $\mathbf{v}_j$ of the solution cone.

NV (INTEGER) First dimension parameter of V(≥ N).

JVEC (INTEGER) Number J of basis vectors of the final cone.

EPS (type according to t) A small parameter which discriminates small quantities against zero, chosen to take into account the accuracy of the machine used.

IOUT (INTEGER)

$= 0$: Gives no intermediate printout,

$= 1$: Gives, for each iteration, the basis vectors of the respective cone, the matrix of scalar products and the index of the inequality taken into account in the next step.

W (type according to t) Two-dimensional array, dimensioned $(\mathtt{MAXV}, \geq \mathtt{M}+1)$, used as working space.

IW (INTEGER) Two-dimensional array, dimensioned $(\mathtt{MA}, 5)$ whose columns serve as book-keepers for certain properties of the system during the iteration procedure.

Method:

The Motzkin-Burger procedure is used to obtain the solution iteratively. Ref. 1 should be consulted before using this subprogram.

Restrictions:

The routine may fail if the matrix **A** is ”ill-conditioned” in a certain sense.

Notes:

A given system of linear homogenous inequalities may have no solution.

Error handling:

Error F500.1: MAXV too small.
Error F500.2: Upper left $\mathtt{N} \times \mathtt{N}$ corner of A is singular.
Error F500.3: Inequality k is inconsistent.
In all cases, a message is written on Unit 6, unless subroutine MTLSET (N002) has been called.

References:

1. K.S. Kölbig and F. Schwarz, A program for solving systems of homogeneous linear inequalities. Computer Phys. Comm. **17** (1979) 375–382.

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