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Approximate Vavilov Distribution and its Inverse

The `VAVLOV` package contains subprograms for fast approximate calculation of functions related to the Vavilov distribution.

For $\kappa > 0$ and $0 \le \beta^2 \le 1$, the Vavilov density function is *mathematically* defined by

$$\phi_V(\lambda;\kappa,\beta^2) \;=\; \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{\lambda s}\, f(s;\kappa,\beta^2)\, ds,$$

where c is an arbitrary real constant and

$$f(s;\kappa,\beta^2) \;=\; C(\kappa,\beta^2)\,\exp\left\{s\ln\kappa + (s+\kappa\beta^2)\left[\ln\left(\frac{s}{\kappa}\right) + E_1\left(\frac{s}{\kappa}\right)\right] - \kappa\,\exp\left(-\frac{s}{\kappa}\right)\right\}.$$

$E_1(x) = \int_0^x t^{-1}\,(1-e^{-t})\,dt$ is the exponential integral, $C(\kappa,\beta^2) = \exp\{\kappa(1+\beta^2\gamma)\}$, and $\gamma = 0.57721\ldots$ is Euler's constant.

The Vavilov distribution function is defined by

$$\Phi_V(\lambda;\kappa,\beta^2) \;=\; \int_{-\infty}^{\lambda} \phi_V(\lambda;\kappa,\beta^2)\, d\lambda$$

and its inverse by $\Psi_V(x;\kappa,\beta^2) = \Phi_V^{-1}(x;\kappa,\beta^2)$.

The function $\Psi_V(x;\kappa,\beta^2)$ can be used to generate Vavilov random numbers (see **Usage**).

Structure:

`SUBROUTINE` and `FUNCTION` subprograms
User Entry Names: `VAVSET`, `VAVDEN`, `VAVDIS`, `VAVRND`, `VAVRAN`
External References: `LOCATR` (E106), `DENLAN` (G110), `DISLAN` (G110)
`COMMON` Block Names and Lenghts: `/G115C1/ 226`

Usage:

```
CALL VAVSET(RKAPPA,BETA2,MODE)
```

sets auxiliary quantities used in `VAVDEN`, `VAVDIS` and `VAVRND`; this call has to precede a reference to any of these entries.

`RKAPPA`	The variable κ (the straggling parameter); ($0.01 \le \kappa \le 12$).
`BETA2`	The variable β^2 (the square of velocity in unit c); ($0 \le \beta^2 \le 1$).
`MODE`	$= 1$; $=$ `0` in the particular case that `VAVDEN` only is referenced after the call to `VAVSET`.

In any arithmetic expression,

`VAVDEN(X)`	has an approximate value of	$\phi_V(\mathtt{X};\mathtt{RKAPPA},\mathtt{BETA2})$,
`VAVDIS(X)`	has an approximate value of	$\Phi_V(\mathtt{X};\mathtt{RKAPPA},\mathtt{BETA2})$,
`VAVRND(X)`	has an approximate value of	$\Psi_V(\mathtt{X};\mathtt{RKAPPA},\mathtt{BETA2})$,

RKAPPA and BETA2 are defined by the last call to VAVSET prior to a reference to VAVDEN, VAVDIS, or VAVRND.

To generate a *set* of Vavilov random numbers with identical κ and β^2, VAVSET should be called once and then VAVRND be referenced repeatedly, using as argument X a random number from a uniform distribution over the interval (0,1).

In any arithmetic expression,

VAVRAN(RKAPPA,BETA2,X) has an approximate value of $\Psi_V(\mathtt{X};\mathtt{RKAPPA},\mathtt{BETA2})$.

To generate *one* Vavilov random number for given values of κ and β^2, VAVRAN should be used, using as argument X a random number from a uniform distribution over the interval (0,1).

VAVDEN, VAVDIS, VAVRND, VAVRAN and X, RKAPPA, BETA2 are of type REAL, and MODE is of type INTEGER.

Method:

The method is discribed in Ref. 1.

Accuracy:

The accuracy depends on the parameters. Although often rather poor from a mathematical point of view, it is usually sufficient for the intended application in physics (see **Notes**).

Restrictions:

No test is made whether the parameters κ and β^2 are in the specified ranges.

Notes:

1. Representing the Vavilov functions by approximations which are both fast and accurate is a difficult task. In view of the requirements in physics, speed is much more important than accuracy. This is taken into account for the present routines.
2. For a more accurate, but much slower, calculation of the Vavilov density and distribution functions, use VVILOV (G116).
3. For $\kappa \leq 0.01$, the Vavilov distribution can be replaced by the Landau distribution (LANDAU (G110)), taking into account that $\lambda_V = (\lambda_L - \ln\kappa)/\kappa$.
4. For $\kappa \geq 10$, the Vavilov distribution can be replaced by the Gaussian distribution with mean $\mu = \gamma - 1 - \beta^2 - \ln\kappa$ and variance $\sigma^2 = (2-\beta^2)/(2\kappa)$.

References:

1. A. Rotondi and P. Montagna, Fast calculation of Vavilov distribution, Nucl. Instr. and Meth. **B47** (1990) 215–224.

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