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Vavilov Density and Distribution Functions

The VVILOV package contains subprograms for the calculation of the Vavilov density and distribution functions. Though generally more accurate, these routines are considerably slower than those in VAVLOV (G115). For $\kappa > 0$ and $0 \le \beta^2 \le 1$, the Vavilov density function is *mathematically* defined by

$$\phi_V(\lambda;\kappa,\beta^2) \;=\; \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{\lambda s}\, f(s;\kappa,\beta^2)\, ds,$$

where c is an arbitrary real constant and

$$f(s;\kappa,\beta^2) \;=\; C(\kappa,\beta^2)\,\exp\left\{s\ln\kappa + (s+\kappa\beta^2)\left[\ln\left(\frac{s}{\kappa}\right) + E_1\left(\frac{s}{\kappa}\right)\right] - \kappa\,\exp\left(-\frac{s}{\kappa}\right)\right\}.$$

$E_1(x) = \int_0^x t^{-1}\,(1-e^{-t})\,dt$ is the exponential integral, $C(\kappa,\beta^2) = \exp\{\kappa(1+\beta^2\gamma)\}$, and $\gamma = 0.57721\ldots$ is Euler's constant.
The Vavilov distribution function is defined by

$$\Phi_V(\lambda;\kappa,\beta^2) \;=\; \int_{-\infty}^{\lambda} \phi_V(\lambda;\kappa,\beta^2)\, d\lambda.$$

Structure:

SUBROUTINE and FUNCTION subprograms
User Entry Names: VVISET, VVIDEN, VVIDIS
Internal Entry Names: G116F1, G116F2
External References: RZERO (C205), RSININ (C336), RCOSIN (C336), REXPIN (C337)
COMMON Block Names and Lenghts: /G116C1/ 322

Usage:

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CALL VVISET(RKAPPA,BETA2,MODE,XL,XU)
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sets auxiliary quantities used in VVIDEN and VVIDIS; this call has to precede a reference to either of these entries.

RKAPPA	The variable κ (the straggling parameter); $(0.01 \le \kappa \le 12)$.
BETA2	The variable β^2 (the square of velocity in unit c); $(0 \le \beta^2 \le 1)$.
MODE	$=$ 0 if VVIDEN is referenced after the call to VVISET; $=$ 1 if VVIDIS is referenced after the call to VVISET.
XL,XU	On exit, XL and XU contain a lower and upper limit as defined below.

In any arithmetic expression,

VVIDEN(X)	has the value	$\phi_V(\mathtt{X};\mathtt{RKAPPA},\mathtt{BETA2})$,
VVIDIS(X)	has the value	$\Phi_V(\mathtt{X};\mathtt{RKAPPA},\mathtt{BETA2})$,

By definition

$$\mathtt{VVIDEN(X)} = 0 \quad \text{if} \quad \mathtt{X} < \mathtt{XL}; \qquad \mathtt{VVIDIS(X)} = 0 \quad \text{if} \quad \mathtt{X} < \mathtt{XL};$$
$$\mathtt{VVIDEN(X)} = 0 \quad \text{if} \quad \mathtt{X} > \mathtt{XU}; \qquad \mathtt{VVIDIS(X)} = 1 \quad \text{if} \quad \mathtt{X} > \mathtt{XU}.$$

RKAPPA, BETA2, XL and XU are defined by the last call to VVISET (with MODE = 0) prior to a reference to VVIDEN, or (with MODE = 1) prior to a reference to VVIDIS.

VVIDEN, VVIDIS and X, RKAPPA, BETA2, XL, XU are of type REAL, and MODE is of type INTEGER.

Method:

The method, based on Fourier expansions, is described in Ref. 1.

Accuracy:

About five significant digits are usually accurate.

Error handling:

Error G116.1: $\kappa < 0.01$ or $\kappa > 10$.
Error G116.2: $\beta^2 > 1$.
These errors can occur when calling VVISET. In both cases, a message is written on Unit 6, unless subroutine MTLSET (N002) has been called.

Notes:

1. Representing the Vavilov functions by approximations which are both fast and accurate is a difficult task. These routines, though rather accurate, are slow. If speed is of higher importance than accuracy, and for calculating Vavilov random numbers, use VAVLOV (G115).
2. For $\kappa \leq 0.01$, the Vavilov distribution can be replaced by the Landau distribution (LANDAU (G110)), taking into account that $\lambda_V = (\lambda_L - \ln\kappa)/\kappa$.
3. For $\kappa \geq 10$, the Vavilov distribution can be replaced by the Gaussian distribution with mean $\mu = \gamma - 1 - \beta^2 - \ln\kappa$ and variance $\sigma^2 = (2 - \beta^2)/(2\kappa)$.

References:

1. B. Schorr, Programs for the Landau and the Vavilov distributions and the corresponding random numbers, Computer Phys. Comm. **7** (1974) 215–224.

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