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Assignment Problem

Subroutine subprogram ASSNDX solves the so-called *Assignment problem*: Given an $n \times m$ matrix A of real numbers $a(i,j)$, find either

1. a set $\{k(1), k(2), \ldots, k(n)\} \in \{1, 2, \ldots, m, 0, \ldots, 0\}$, where $0, \ldots, 0$ indicates $\max(n-m, 0)$ zeros, and where for non-zero elements $k(p) \neq k(q)$ for $p \neq q$, which minimizes

$$S = \sum_{i=1}^{n} a(i, k(i))$$

   assuming that $a(i,0) = 0$, or

2. a set $\{k(1), k(2), \ldots, k(m)\} \in \{1, 2, \ldots, n, 0, \ldots, 0\}$, where $0, \ldots, 0$ indicates $\max(m-n, 0)$ zeros, and where for non-zero elements $k(p) \neq k(q)$ for $p \neq q$, which minimizes

$$S = \sum_{j=1}^{m} a(k(j), j)$$

   assuming that $a(0,j) = 0$.

Structure:

SUBROUTINE subprogram
User Entry Names: ASSNDX
Files Referenced: Unit 6

Usage:

```
CALL ASSNDX(MODE,A,N,M,IDA,K,SMIN,IW,IDW)
```

MODE (INTEGER) Must be set either 1 (for case (1)), or 2 (for case (2)).

A (REAL) Two-dimensional array of dimension (IDA, $\geq$ M). Must contain, on entry, the matrix A. Destroyed during execution.

N (INTEGER) Number n of rows of A.

M (INTEGER) Number m of columns of A.

IDA (INTEGER) Declared first dimension of A in the calling program (IDA $\geq$ N).

K (INTEGER) One-dimensional array of length $\geq \max$(N, M). Contains, on exit, the assigned set of integers $\{k(1), \ldots, k(n)\}$ or $\{k(1), \ldots, k(m)\}$, respectively.

SMIN (REAL) The calculated minimum value of S.

IW (INTEGER) Two-dimensional array of dimension (IDW, $\geq$ 6). Used as working space.

IDW (INTEGER) Declared first dimension of IW in the calling program (IDW $\geq \max$(N, M)).

Method:

The subprogram is based on the Algol procedure given in Ref. 3.

Error handling:

Error `H301.1`: $\mathtt{N} < 1$ or $\mathtt{M} < 1$.
A message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.

Examples:

The following example illustrates a possible use of the subprogram. A workshop has to carry out N jobs, each of which can be performed on any of $M(> N)$ available machines. The cost of performing job I on machine J is $A(I, J)$. It is required to assign jobs to machines in such a way as to minimize the total cost. The solution is obtained by calling the subprogram with $\mathtt{MODE} = 1$ and then assigning job I to machine $\mathtt{K(I)}$, $(\mathtt{I} = 1, 2, \ldots, \mathtt{N})$.

References:

1. J. Munkres, Algorithms for the assignment and transportation problems, J. SIAM 5 (1957) 32–38.
2. R. Silver, An algorithm for the assignment problem, Comm. ACM **3** (1960) 605–606.
3. R. Silver, Algorithm 27 ASSIGNMENT, Collected Algorithms from CACM (1960).

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