LOREN4	CERN Program Library	U101

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Language : Fortran

Library: KERNLIB
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Lorentz Transformation

This routine transforms momentum and energy of a particle from one Lorentz-frame to another.
Seen from the reference system Σ, the other system Σ' has the velocity $\vec{\beta}$, with $\vec{\eta} = \gamma\vec{\beta}$.
If a rest mass M is tied to system Σ', with energy E and momentum $\vec{P}$, we have:

$$\vec{\beta} = \vec{P}/E, \qquad \vec{\eta} = \vec{P}/M, \qquad \gamma = E/M.$$

The momentum and energy of a particle with mass m is

in system Σ : $\vec{p}$ and $e = \sqrt{p^2 + m^2}$,
in system Σ' : $\vec{p'}$ and $e' = \sqrt{p'^2 + m^2}$.

Structure:

`SUBROUTINE` subprogram
User Entry Names: `LOREN4`

Usage:

```
CALL LOREN4(S,A,X)
```

with the 4–vectors $\mathtt{S} = (\vec{P}, E)$ and $\mathtt{A} = (\vec{p}, e)$ calculates the transformed 4–vector $\mathtt{X} = (\vec{p'}, e')$.
`LOREN4` contains one square-root to derive M from P and E.

Method:

If we split $\vec{p} = \vec{p}_L + \vec{p}_T$ into components parallel and normal to $\vec{\beta}$, where

$$\vec{p}_L = \frac{\vec{p}\,\vec{\eta}}{\eta^2}\,\vec{\eta}, \qquad \vec{p}_T = \vec{p} - \vec{p}_L,$$

we can write the transformations as

$$\vec{p'}_L = \gamma\,\vec{p}_L - \vec{\eta}\,e, \qquad \vec{p'}_T = \vec{p}_T, \qquad e' = \gamma\,e - \vec{\eta}\,\vec{p}$$

and get

$$\begin{aligned}
\vec{p'} &= \vec{p} + (\gamma - 1)\vec{p}_L - e\,\vec{\eta} \\
&= \vec{p} + \vec{\eta}\,((\gamma - 1)\vec{p}\,\vec{\eta}/\eta^2 - e) \\
&= \vec{p} + \vec{\eta}\,(\vec{p}\,\vec{\eta}/(\gamma + 1) - e) \qquad \text{(because of} \quad \eta^2 = \gamma^2 - 1) \\
&= \vec{p} + \vec{P}\,(\vec{p}\,\vec{P}/(E + M) - e)/M, \\
\vec{e'} &= \gamma e - \vec{\eta}\,\vec{p} \\
&= (eE - \vec{p}\,\vec{P})/M.
\end{aligned}$$

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