Author(s) : K.S. Kölbig **Library:** MATHLIB
Submitter : **Submitted:** 15.02.1989
Language : Fortran **Revised:** 01.12.1994

Beta-Term in Wigner's D-Function

Function subprograms `RDJMNB` and `DDJMNB` calculate the β-term $d^j_{mn}(\beta)$ in the matrix element of the finite rotation operator (Wigner's D-function)

$$D^j_{mn}(\alpha,\beta,\gamma) \;=\; e^{-im\alpha}\, d^j_{mn}(\beta)\, e^{in\gamma}$$

by using the formula (Ref. 1, No. 4.3.1(3))

$$d^j_{mn}(\beta) \;=\; (-1)^{j+m}\sqrt{(j+m)!(j-m)!(j+n)!(j-n)!}\;\times$$
$$\sum_k (-1)^k\,\frac{\cos^{2k-m-n}(\frac{1}{2}\beta)\;\sin^{2j+m+n-2k}(\frac{1}{2}\beta)}{k!(j+m-k)!(j+n-k)!(k-m-n)!}$$

for arbitrary (either all integer or all half-integer) values of j, m, n such that $j \geq 0, |m| \leq j$ and $|n| \leq j$. The summation over k runs from $\max(0, m+n)$ to $\min(j+m, j+n)$.
On computers other than CDC or Cray, only the double-precision version `DDJMNB` is available. On CDC and Cray computers, only the single-precision version `RDJMNB` is available.

Structure:

`FUNCTION` subprograms
User Entry Names: `RDJMNB`, `DDJMNB`
Obsolete User Entry Names: `DJMNB` $\equiv$ `RDJMNB`
Files Referenced: `Unit 6`
External References: `MTLMTR` (N002), `ABEND` (Z035)

Usage:

In any arithmetic expression,

`RDJMNB(AJ,AM,AN,BETA)` or `DDJMNB(AJ,AM,AN,BETA)` has the value $d^j_{mn}(\beta)$,

where `AJ` $= j$, `AM` $= m$, `AN` $= n$ and `BETA` $= \beta$. `RDJMNB` is of type `REAL`, `DDJMNB` is of type `DOUBLE PRECISION`, and `AJ`, `AM`, `AN`, `BETA` have the same type as the function name. `BETA` has to be given in degrees.

Restrictions:

`0` $\leq$ `AJ` $\leq$ `25`, $|$`AM`$|$ $\leq$ `AJ`, $|$`AN`$|$ $\leq$ `AJ`, `0` $\leq$ `BETA` $\leq$ `360`.

Accuracy:

Approximately full single- or double-precision machine accuracy, at least for small values of the indices.

Error handling:

Error `U501.1`: If any of the restrictions is not satisfied, the function value is set equal to zero, and a message is written on `Unit 6`, unless subroutine `MTLSET` (N002) has been called.

References:

1. D.A. Varshalovich, A.N. Moskalev and V.K. Khersonskii, Quantum theory of angular momentum, (World Scientific, Singapore 1988) 76

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